MODELING AND SIMULATION OF PATIENT FLOW IN HOSPITALS FOR RESOURCE UTILIZATION

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Abstract

Hospital costs play a significant role in national budgets. To some degree, patients are suffering from lack of vacant beds and caretakers. Emergency Department (ED) crowding causes a series of negative effects, e.g. medical errors, poor patient treatment and general patient dissatisfaction. One road to improve the typical clinical system is to describe the patient flow in a model of the system and how the system is constrained by available equipment, beds and personnel.

This paper focuses on modeling and simulation of the capacity of utilities and how using advanced control techniques can enable intelligent scheduling, leading to smooth patient flow to reduce emergency department crowding. By comparing different models, the most efficient ones will be identified for implementation. The idea is that hospitals can use the proposed models to predict the future resident patient number in each department/ward. The caretakers can use the predicted results with other information to make decisions of admission of the intake patients, find the optimal pathway for the patients to minimize the residence time, and make intelligent scheduling to reduce the queueing length in the hospital.

1. Introduction

Hospital cost plays a significant role in national budgets. To some degree, patients are suffering from lack of vacant beds and caretakers. Bottlenecks and congestion are everyday business. The probability of unacceptable refused admission is around 14% [1].

Emergency Department (ED) crowding causes a series of negative effects e.g. medical errors, poor patient outcomes and patient dissatisfaction. Patient satisfaction, staff satisfaction, and hospital revenue are all negatively impacted when patients, information, and materials do not move through hospitals in a timely and efficient way [2].

The hospital crowding is primarily regarded as the consequence of inadequate medical resources. However, recent research has shown that the highly stochastic process of incoming patients causes the violation of resources, which would lead to such crowding. Hence, to simply expand medical care capacity may do little to relieve the emergency department crisis. The situation can be potentially improved by optimizing the utilization of medical resources, e.g. bed, equipment and personnel

To be able to improve the typical clinical system, it is necessary to develop the patient flow model through the system and describe how the system is constrained by available equipment, beds and personnel. Queuing Theory with Markov Chain (QTMC), and Discrete Event Simulation (DES), are the methods that are used to describe the system.

The first model (QTMC) is only able to consider limited scenarios that can occur. One published QTMC model of the orthopedic department of the Middelheim hospital focuses on the impact of outages of the personnel (preemptive and non-preemptive outages), on the effective utilization of resources, and on the flow time of patients [3]. Several queuing network solution procedures are developed such as the decomposition and Brownian motion approaches.

On the other hand, DES has been well recognized in healthcare. These models are broadly used for the validation of other models. The DES models offer a valuable tool to study the trade-off between the capacity structure, sources of variability and patient flow times [4].

This paper is organized as follows:

In section 2, the definition of patient flow in the hospital is introduced. Diagnosis and treatment of patients and uncertainty in the system is discussed. In section 3, several modeling techniques will be described. Two modeling process, the Queueing Theory and Markov Chain (QTMC) model, and Discrete Event Simulation (DES), are applied to describe the patient flow. In section 4, possibilities of using Model Predictive Control (MPC) optimize the patient flow is discussed. In Section 5, conclusions are drawn.

2. Patient flow

The patient flow can be considered as the movement of patients through a set of locations in a healthcare facility. There are six characteristics of patient flow. These characteristics are the basic elements and assumptions in the patient flow model [5].

- Long waiting lists with respect to complex operations
- Uncertainty and apparent chaos are common
- Every patient is unique
- Relative large variation in Length of Stay (LoS)
- The incidence of complications
- Emergency admissions

The patient flow can be considered as a combination of physical flow, information flow and decision flow:

- Physical flow: In this view, the flow of all the existing materials e.g. patients, test/treatment materials, or caretakers is considered. Some examples are patient pathway, transport of the blood, or the flow of caretakers.
- Information flow: Include information about the patients and the states in different departments, such as the test results, the occupancy of beds, waiting lists of operation departments, numbers of doctors and nurses who are available, etc.
- Decision flow: The decision of a different pathway of physical flow or information flow is the decision flow. The decision flow depends on the diagnosis of the patient and the state in the hospital. Sometimes, decision flow can be a part of information flow.

The components of intake emergency patients are patients from their home, other institutions, private care, other wards, site of the incident or born in hospital. The different sources have different inter-arrival rate; the combination of different intake patients presents a certain distribution. This distribution is able to be predicted by analyzing the history data.

All the intake patients at hospitals can be classified into two modes based on the sick level: emergency patients and planned patients. The queueing policy will be different based on the illness level, such as without other factors, a patient with an open wound has a higher priority over a patient with a stomach pain. As a consequence, patients with lower priority have to wait longer. Arrival patients follow a process as depicted in Fig. 1.

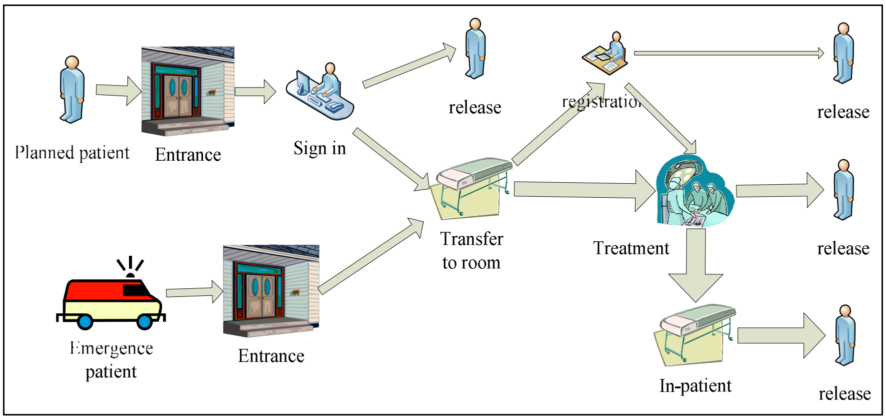

Fig. 1 Patient flow of the emergency department

The arriving patient received by the registration clerk who records patient arrival time and the symptoms. The nurse checks the records and determines the acuity of the illness. If patients arrive in an ambulance, they begin their process at the ER bed area, with the registration paralleling with emergency care service.

In general, after the acuity of the illness has been determined, the patient either goes to the bed area or queues for a bed with a priority queue discipline. Once the patient gets an available bed, the medical treatment begins. First, the necessary tests are ordered. After the results are obtained from the laboratory, the caretakers will decide whether the patient needs to be admitted or not. The admitted patients will continue treatment and are distributed to another care unit. The records and all the test results will follow the patients to the other units. Other patients will receive necessary therapeutic care and are sent home. The records will cease to exist.

Being admitted to the hospital, being discharged from the ED, leaving the emergency department before treatment, and deceased are the four ways a patient may exit the ED.

The following factors influence the results of diagnosis and selection of treatment [6, 7, 8].

- Priority of patients
- Quantity of physicians and nurses
- beds in the wards
- Treatment equipment
- Location of different departments in the hospital.

Some real data from Ringerike sykhus (Norway) are provided by IMATIS AS [19]. These data include 513 samples. Each sample records the patent ID, the arrival time, departure time, the ward to enter, next ward to enter after treatment. In Ringerike sykehus, different departments share the same wards, and the wards are classified by the location and facilities. The wards include AK, MO, K2,3,4, J2,3,4, L2,3,4, IN, I3, 4, and so on [9]. Each ward has a capacity of 9 beds. One typical pathway is shown in Fig. 2. Fig. 2 also indicates the frequancy of patients going from the AK ward to one of the other ward. Fig.3 shows the number of patients in AK ward at a given time.

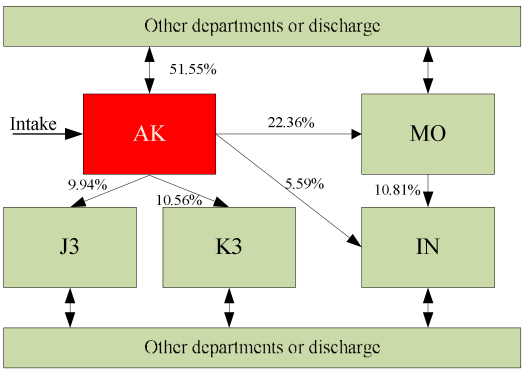

Fig. 2 Selected patient flow at Ringerike hospital

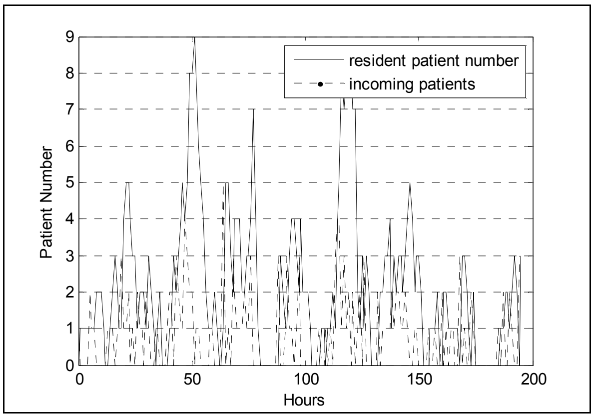

Fig. 3 Resident patient number in AK at Ringerike hospital

The residence time, is the time when the patients arrival at the hospital until they come out of the hospital. The residence time in the hospital includes the residence time at each department and the transfer time. For a department, the residence time includes the waiting time, and the processing time (service time). The residence time in a process following is considered as an Exponential distribution. In actual processes, the distribution can be studied from the history data.

The mean value of arrival rate is influenced by many factors. There is a big difference between weekdays and weekends, working time and resting time, and so on. Big events may also increase the number of patients, e.g. anniversary, sports event, traffic accidents etc. In a day more patients arrive at the hospital during the day and evening than the morning.

The planned patients make an appointment with the hospital. The patients will come to the hospital based on a schedule, which also means this variable can be controlled.

To analyse stochastic variables, the corresponding distributions of interarrival rate and residence time should be found. Herein, Exponontial distribution, Weibull distribution, and Poission distribution are inverstageted. The Weibull distribution has a flexible shape. This distribution has been used successfully in many applications as a purely empirical model [17]. The Exponential distribution has only one unknown parameter, This distribution has a memoryless property, which means previous states don't influence the future states [18]. If the variable in the Exponential distribution is integer, the variable can be expressed by Poisson distribution. Poisson distribution has the same properties with Exponential distribution.

For Matlab, functions e.g. '`wblfit`', '`expfit`', '`poissfit`', etc. in the Statistical Toolbox can be used to estimate the parameters of different distributions. One example to obtain the parameters of Exponential distribution is as follows:
Given the data X, `lambda = poissfit(X)` returns the maximum likelihood estimate of the parameter corresponding to 95% (default) confidence intervals of the Poisson distribution, λ [18].

The AK ward mainly processes primary care of the emergency patients. Comparing different distributions with the real data, the Weibull distribution fits the real data the best (mean and variance values in Fig. 4, Tab. 1). The Weibull distribution is suitable to demonstrate the properties of the residence time in the AK ward. The resident patient number from the data is always less than the capacity of 9 beds (Fig. 3). Thus, it is seldom that patients have to queue for beds in the samples. In this situation, the residence time can be treated as the service time. The parameters in Tab. 1 are estimated using the Matlab Statistical Toolbox.

Tab. 1 Residence time distribution in AK at Ringerike hospital

Distribution:	Mean:	Variance:	Parameters:
Weibull	2.54	2.49	a 2.85 b 1.65
Exponential	0.82	0.68	μ 0.83

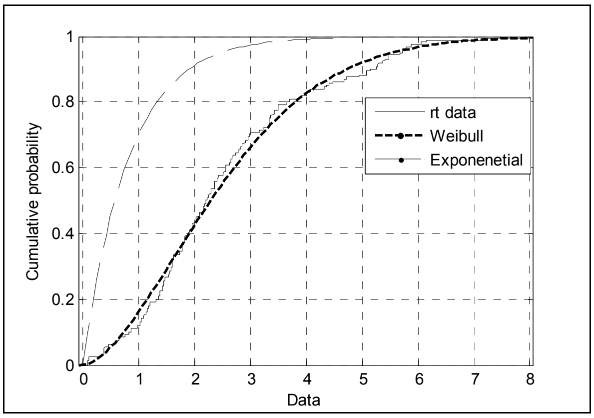

Fig. 4 Residence time distribution in AK at Ringerike hospital

The wards (MO, K3, IN) mainly process treatment after primary care, they have different functions and are mainly classified by their location in the hospital. The residence time in these wards are much longer than that of the AK wards (Tab. 2). By analyzing the real data, all the distributions of the residence time of these wards can be treated as an Exponential distribution. In the wards MO, FO, K3, and IN, the resident patient number of selected data is less than the capacity of each ward. The residence time can be taken as the service time.

Tab. 2 Selected residence time distributions in each ward

Wards	Distribution	Parameters
AK	Weibull	a 2.85 b 1.65
MO	Exponential	μ 15.56
K3	Exponential	μ 52.02
IN	Exponential	μ 34.37

The incoming patient number is a discrete stochastic variable (see dotted line in Fig. 3). The Poisson distribution fits the discrete stochastic variables well. The parameters of all these 5 wards are shown in Tab. 3. The AK ward has the highest arrival frequency of about 0.8 patients per hour. The high arrival rate and the short service time (high throughput) lead to a big variation in performance during one day. On the other hand, the average time of one patient arrival at the other departments is more than 4 hours. The patient number has less variance during one day.

Tab. 3 Incoming patient number distribution

Distribution (Poisson)	AK	MO	FO	K3	IN
Parameter λ	0.82	0.17	0.23	0.087	0.095

3. Simulation of the patient flow

3.1 Discrete Event Simulation

In DES, the operation of a system is represented as a chronological sequence of events. Each event occurs at an instance in time and marks a change of state in the system [10]. The modeled system is dynamic and stochastic. DES includes Clock, Events List, Random Number Generators, Statistics and Ending Condition [11].

For example, in the process that patients wait for a bed in the ward, the system states are queueing length or number of vacant beds. The system events are patients-arrival and patients-departure. The system states, like vacant beds are changed by these events. The random variables that need to be characterized to model this system stochastically are patient arrival time and residence time. To simulate such system, first generate a series of random entities based on the distribution. Let (n,t) be n patients coming into the station at time t. Then all the incoming patients during $(t_1, t_2 = t_1 + dt, t_3 = t_2 + dt, ..., t_k)$ can be expressed as $\{(n_1, t_1), (n_2, t_2), ..., (n_k, t_k)\}$. Here n_1, n_2, …, n_k are random numbers. dt is constant. The simulator generates service rate for each patient, l_1, l_2, …, l_k which are random numbers. All the random numbers obey a certain distribution. The patients leave the ward when the residence time is over. The simulator stores all the data. The patient number and other results can be obtained by analyzing the saved data. Such as to compute the resident patient number at time t_i, the simulator find out the patients that time t_i is between this patients' arrival and departure time. The number of these patients is the resident patient number (Fig. 5).

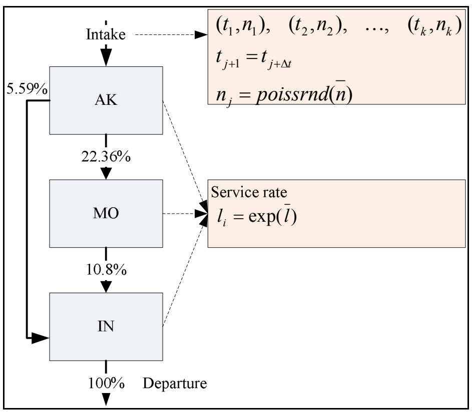

Fig. 5 DES of patient flow at Ringerike hospital

The simulation logic of DES is shown in Fig. 6. Here in order to get the mean value of the results at each time point, DES is repeated (first loop).

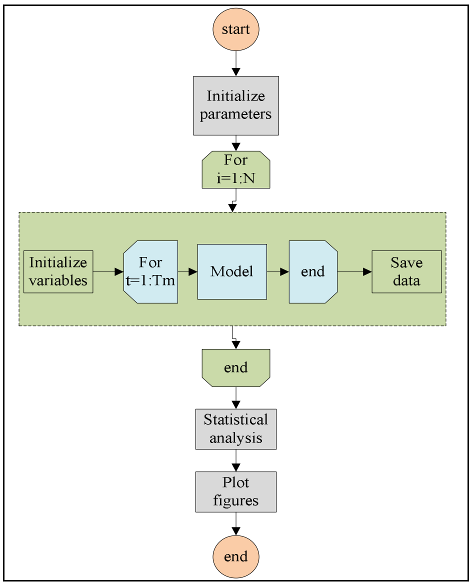

Fig. 6 DES programming logic

There are several advantages to build such models [12].

- Detailed system behavior can be modeled;
- It is possible to model the performance, dependability;
- Less matrices computing.

Also there are some drawbacks compared with other models [12].

- Long execution time;
- Simulation results are difficult to interpret;
- It is quite likely that some rare events or states are never encountered by the simulation runs.

3.2 Queuing Theory and Markov Process

If a queueing system has m beds, a Poisson distributed incoming rate and an Exponential distributed service rate, this queueing system can be denoted by M/M/m [17].

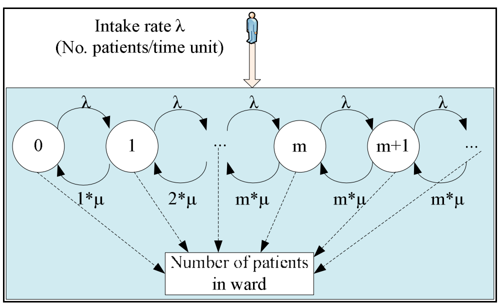

Fig. 7 QTMC model of one ward

In Fig. 7 :

μ the service rate in a station, the average time a doctor spent on a patient.

λ the inter arrival rate, is the input.

Define $\pi_k(t)$ as the transient state probability vector is the state. An M/M/m queueing system can be described by the model (Fig. 7) [10].

$1 \le k \le m$:

$$\frac{d\pi_k(t)}{dt} = -(\lambda + k\mu)\pi_k(t) + \lambda\pi_{k-1}(t) + k\mu\pi_{k+1}(t)$$

$k > m$:

$$\frac{d\pi_k(t)}{dt} = -(\lambda + m\mu)\pi_k(t) + \lambda\pi_{k-1}(t) + m\mu\pi_{k+1}(t)$$

Boundary model $k = 0$:

$$\frac{d\pi_0(t)}{dt} = -\lambda\pi_0(t) + \mu\pi_1(t)$$

This system of ODEs (Ordinary Differential Equation) can be written in Matrix form as:

$$\frac{d\pi(t)}{dt} = Q\pi(t), \pi(0) = (\pi_0(0), \pi_1(0), \ldots) \qquad (1)$$

where:

$\pi(t) = (\pi_0(t), \pi_1(t), \ldots, \pi_m(t))^T$, m is the number of states. $\pi_0(t)$ is the boundary. Since $\pi(t)$ is a probability vector, the sum of the states equals to 1.

$$1^T{}_{m+1}\pi = 1 \qquad (2)$$

The rank of Q is m-1. Q is not a singular matrix. In order to compute the value of $\pi(t)$, substitute Eq. (2) into Eq. (1), then the differential equation becomes:

$$\frac{dx}{dt} = Q_q x + B_1 u'$$

where $u' = \lambda$ the intake rate can be varied.

Here x is the state, $x = \pi_{0:m-1}$, $\dim(x) = m$

$$\frac{d}{dt}(1^T\pi) = 1^T Q\pi \xrightarrow{1^T\pi=1} 1^T Q\pi = 0;$$

Q_q can be written as a function of inputs u' :

$$Q_q = A_1 u_3 + A_2;$$

$$A_1, A_2 \in R^{(m-1)(m-1)}.$$

$$A_1 = \begin{bmatrix} -1 & 0 & \cdots & & & 0 \\ 1 & -1 & \ddots & & & \vdots \\ 0 & 1 & \ddots & & & \\ \vdots & \ddots & \ddots & & & \\ 0 & \cdots & 0 & 1 & -1 & 0 \\ -1 & -1 & \cdots & -1 & 1 & -2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & \cdots & & & & 0 \\ 0 & -2\mu & 2\mu & & & & \vdots \\ \vdots & & -3\mu & 3\mu & & & \\ & & & \ddots & \ddots & & \\ & & & & -S\mu & S\mu & \\ & & & & \ddots & \ddots & 0 \\ & & & & 0 & & S\mu \\ 0 & \cdots & & & 0 & S\mu & -S\mu \end{bmatrix}$$

u' can be separated into two categories; which are generated by emergency patients and planned intake patients. The Emergency patients are uncontrollable and can be regarded as disturbance v; the planned intake patients can be scheduled, and can be taken as the control variable u in this model.

$$u' = u + v$$

$$\frac{dx}{dt} = Q_q x + B_1(u+v) = (A_1(u+v) + A_2)x + B_1(u+v)$$

The patient number y can be approximately computed by:

$$y = Dx,\ D = [0,1,2,...,m-1]$$

To obtain the solution for queuing networks (Fig. 5), first introduce one property of Poisson streams.

Different streams of patients may come into one department. If the streams obey the Poisson distribution, then joining these Poisson streams produces a single Poisson stream. Patients will distribute to different departments after one treatment. If the patient number obeys the Poisson distribution, probabilistically splitting this stream gives rise to two or more Poisson streams [12]. These two properties guaranteed the patients number coming out from AK ward to IN or MO ward, and the combination of patients from AK and MO wards to IN ward also obeys Poisson distribution.

The outputs of each station ($\lambda_{o,1}, \lambda_{o,2}, \lambda_{o,3}$) is computed by solving the 2nd order equation. This is an approximate value obtained from the steady state solution of a queueing system (M/M/m). The output of the queueing system is only dependent on the current input. Here if the resident patient number has not reached the steady state, the current input is treated as it has reached the steady state with another input λ_o. This value is only used to compute the departure patient number [10].

$$L = \frac{\lambda_o / S + \lambda_o * C1/(S*\mu)}{1 - \lambda_o/(S*\mu)}$$

$$\Rightarrow \lambda_o = \frac{1}{2}\mu[S + C1 + L - \sqrt{(S^2 + 2SC1 - 2SL + C1^2 + 2C1L + L^2)}\ ]$$

Here, The current patient number L, the number of servers S, and current intake patient rate μ are known. $C1$, which equals to the probabilities that the patients must queue for a bed (including full beds occupied situation), can be computed in real time.

The inputs of each station are the output streams from other stations multiply the probabilities of patients from other stations flowing into this station.

The model in a three station queueing network (Fig. 5) can be written as

$$\frac{dx'}{dt} = (A_1'(u'+v') + A_2')x + B_1'(u'+v')$$

$$\frac{dx''}{dt} = (A_1''(u''+v'') + A_2'')x'' + B_1''(u''+v'')$$

$$\frac{dx'''}{dt} = (A_1'''(u'''+v''') + A_2''')x''' + B_1'''(u'''+v''')$$

$$y' = D'x'$$

$$y'' = D''x''$$

$$y''' = D'''x'''$$

Here

$$v' = \lambda_1$$

$$v'' = \lambda_2 = p_{12} * \lambda_{o,1} + p_{32} * \lambda_{o,3}$$

$$v''' = \lambda_3 = p_{13} * \lambda_{o,1} + p_{23} * \lambda_{o,2}$$

$$\lambda_{o,1} = \frac{1}{2}\mu_1[S_1 + C1' + y' - \sqrt{(S_1^2 + 2S_1C1' - 2S_1y' + C1'^2 + 2C1'y' + y'^2)}\ ]$$

$$\lambda_{o,2} = \frac{1}{2}\mu_2[S_2 + C1'' + y'' - \sqrt{(S_2^2 + 2S_2C1'' - 2S_2y'' + C1''^2 + 2C1''y'' + y''^2)}\ ]$$

$$\lambda_{o,3} = \frac{1}{2}\mu_3[S_3 + C1''' + y''' - \sqrt{(S_3^2 + 2S_3C1''' - 2S_3y''' + C1'''^2 + 2C1'''y''' + y'''^2)}\ ]$$

$$C1' = E'x'$$

$$C1'' = E''x''$$

$$C1''' = E'''x'''$$

The simulation logic is shown in Fig. 8

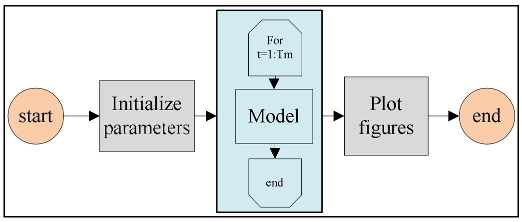

Fig. 8 Queueing theory programming logic

Fig. 9 is the simulation results of patient number and Fig. 10 is the intake patient number in AK, MO, and IN wards. The plots of DES and MCQT are the results simulated by the two models (DES, and MC + QT). The intake patient number has the circle time of eight hours. From Fig. 3, we can see the intake patient number have cyclical changes, and the cycle time is around 8 hours. The selection of 8 hours circle time fits the actual situation and previous studies [2]. The MCQT model is less sensitive than the DES model, the fluctuation of MCQT model is less than the DES model, when the inputs have great changes.

The two plots DES-S and MCQT-S show the patient number of two models when the intake patient number and service time is constant. The relationship of AK, MO, and IN wards is shown in Fig. 5. The plots show that both models can simulate the patient number and they have similar results.

The state space of MCQT is 100 and the ensemble size of DES is 200 (first loop in Fig. 6). The larger the dimension of the state space the better results can be obtained. But a large state space requires more computation memory and time to simulate. The results show that the largest average patient number is less than 15. The number of state space with 100 is possible to get a very accurate result.

The DES plots have more fluctuation between each simulation plots. These unstable properties can be improved by increasing ensemble size, but a large ensemble size of simulations will lead to a higher computation cost.

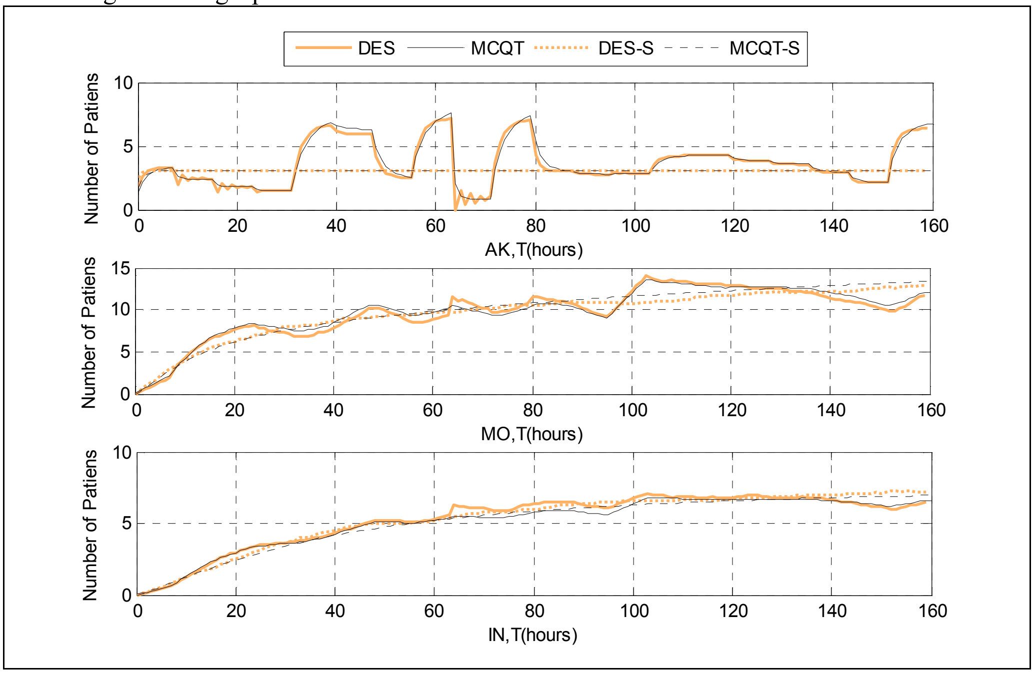

Fig. 9 Patient number (AK, MO, IN)

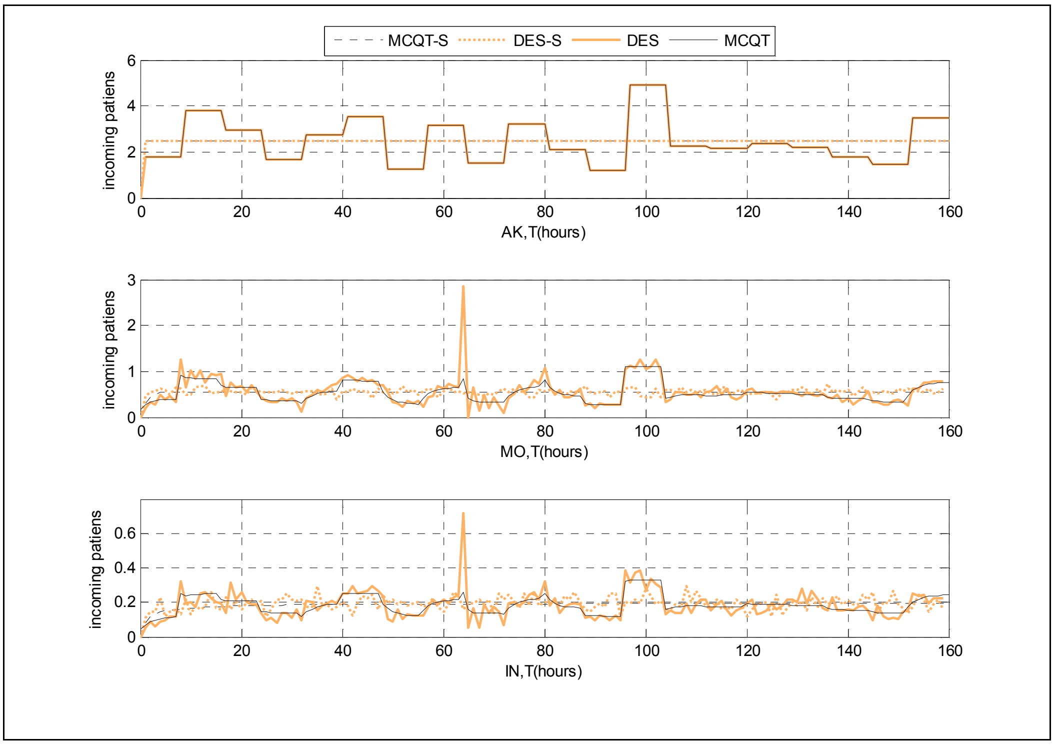

Fig. 10 Intake patient number (AK, MO, IN)

4. Optimal control of patient flow

In order to perform the function of the emergency department and the other departments optimally, we can control some variables, e.g. the work schedule of caretakers, the available beds, and the incoming patients. The control variable here is chosen as the planned incoming patients.

Many algorithms have been studied to control the queuing systems, e.g. dynamic programming [14], and Lagrange approach of adaptive control based on Markov Chain model [15]. In this chapter, the use of Model Predictive Control is discussed.

The model predictive controller uses the model and current measurements to calculate future inputs that will fulfill the objects and variable constraints. Model predictive controllers rely on dynamic models of the process. The models are used to predict the future unknown variables.

The dynamic system can be modeled as or be transformed to a linear state space model. Then the effect of changes in unknown variables can be added together to predict the response. This approach leads the control problem to a series of matrix algebra calculations that are fast and robust [16].

In the queueing system here, the predictive model is based on queuing theory and the Markov Chain model. The inputs are either planned or emergency patient inter-arrival rate. The object is to reduce the queuing length in a certain time horizon. The measurements are the current patient number in the system.

A control objective J_k (or cost function) is a measure of the process behavior over the prediction horizon L. This function can be the difference between future outputs and some specified future reference, and sometimes recognizing that the control is also costly. This objective is minimized with respect to the future control inputs and only the first control input is actually used for control. This optimization process is solved again at the next time instance. The advantage of MPC is that constraints of the process variables can be treated in a simply way. In order to optimize the patient number and minimize the transport of patients, both the deviation between outputs and references, and the variation of the control variable should be considered. The intake patient number should be positive. These algorithms implemented real-time is shown in Fig. 11.

The states are the probabilities x which cannot be measured. The inputs are patient arrival rate which is estimated. Current patient number y can be measured. In order to use MPC algorithm, the states should be estimated. One simply way is to set the current state $x_t(i)=1, x_t(n \neq i)=0$, when there are i-1 patients in the wards. A state estimator (e.g. Kalman Filter) can also be used to estimate the state.

The patient number has a daily and weekly cycle of change. When the throughput of the ward is high, the prediction horizon can be 1 day. Otherwise, the horizon can be 1 week. A smaller time step requires larger memory. The time step T could be selected as 1 hour. This selection can present nature of the system, and also be easy to compute and control. Other selections with 6 or 8 hours time step are preferred for high throughput systems or long prediction horizon. The data from Ringerike sykhus shows that the throughputs of the wards are low and the data varies per day significantly. The MPC algorithm with 1 hour Time step and 24 hours horizon can be selected.

The states number combined with the unknown variables and horizon may cause large matrix and complex matrix computing. This algorithm is time consuming. Compared with the time step, the long computing time is tolerable.

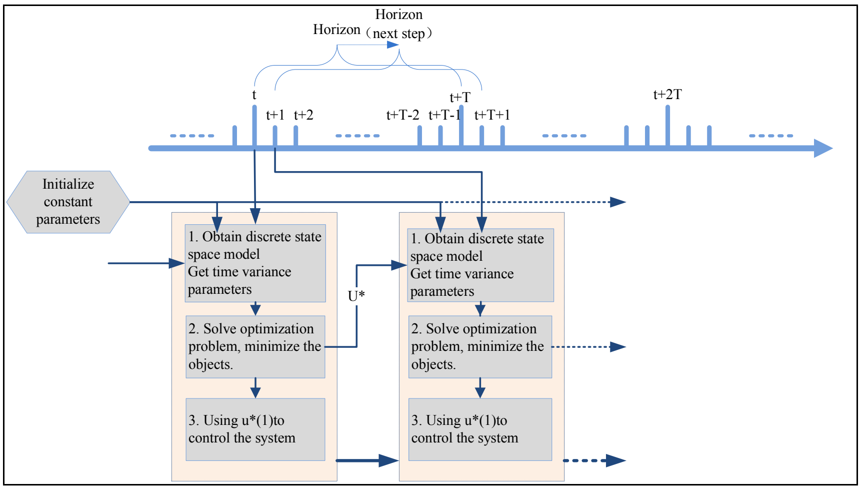

Fig. 11 MPC for periodic variance system

5. Conclusions and future works

Health care resources and patient treatment have become increasingly important and expensive. The task of balancing the delivery of quality health care and the facilities in the hospitals is becoming a hot topic.

Patient flow is the transport of the patients through the health care system, including three different phases: the physical flow, information flow and decision flow. In order to find some possible solutions for improving the patient flow and predicting the number of the patient in different departments, models based on different theories, e.g. the Queuing Theory and Markov Chain model, the DES are introduced and compared.

The simulation based on Queuing Theory and Markov Chain can be a good approach for implementation in the real hospital. This model provides a quite good method to handle the randomness and uncertainty in the patient flow, but it is not easy to find a proper mathematical model when the process is complex. The DES models are commonly used in checking the other models. The approach of using this model in patient flow also gives a quite realizable result.

The patient flow can be controlled by a host of different methods. The MPC methods described here are adapted from its use in process control. And because of the limitation of the control variable, improved algorithms, e.g. more efficiently handling the integral variables and the constraints, should be further developed.

The future work can be considered in the following aspects:

- Modeling systems with more complex properties.

The real process is more complex than the case studies in this thesis. Problems like processes with different service disciplines, and process which includes the effect of caretakers can be modeled in the future.

- Optimal control of the queuing system

Section 4 demonstrated the possibilities of using MPC to control the queueing systems. More research should be done to get the theory to practice.

- Reduce and optimize the matrices computing.

Simulator generates plant of matrices during simulation. These matrices are time consuming and will take up lots of resources. Methods to reduce and optimize the matrices computing can be investigated in the future.

- Implementation in the real hospital.

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