

# Discontinuous Simulation Techniques for Worm Drive Mechanical Systems Dynamics

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## Abstract

In this paper we consider application of Two-Sided Wedge Mechanism (TSWM) with clearance and friction contact as Design Dynamic Model of Worm Drives Machines. The Mathematical model formulation based on partitioning co-ordinate method for identify kinematic and force constraints together with dynamic equations in closed loop form. The residual equation of motion holds Discontinuities and switching conditions for simulation contact transitions and Force Transfer Function according to system state. The simulation results are based on switching algorithm which realised step by step integration scheme and control procedure which implemented in Matlab ODE programs.

## Introduction

The discontinuities have presence in constrained Multibody System Dynamics when contact forces relations between some moving machine parts may be instantaneously changes at time depending on systems state and simulation parameters. Contact-cases modification result in discontinuities of corresponding constraints equations undergoing structure-variant modelling is given in literature [1,2,3,4]. The mathematical formulation of such Multibody System (MBS) in residual form have been presented by ODEs with Discontinuities in the right-hand side. The systems integration scheme based on concept of switching conditions in order to monitored and obtains time points when discontinuities are present in an integration step. Different aspects and approaches of this numerical problem are well discussed in [5,6].

The force constraints problem undergoing discontinuity analysis is very actually for Worm Drives Mechanical System, which use as object of simulation in these paper. It is well know that Worm gearing has won wide acceptance for

industrial drives because of its many advantages of conjugate tooth action, compactness, high-speed reduction ratios and load-carrying capacity. Since meshing action between worm thread and the teeth of the driven worm gear is predominantly sliding where sliding friction forces in variable closed loop contact-cases depends from value and direction of sliding velocity that have great influence on result efficiency of force transmission. By presence of clearance, which has usually manufacturing and constructive nature for provides the relative mobility of gearset parts we can explain the transitions between different contact-cases indicated by discontinuities in the internal force contour. This situation poses the problem of how to take advantages of Multibody Dynamics and discontinuous approach to develop effective Design and Mathematical Model for simulation mechanical efficiency of the Worm Drives Unit.

Due to the worm geometry that is of screw thread configuration and orthogonal cross-axis of assembly motion we propose to consider the Two-Sided Wedge Mechanism (fig.1) as Design Dynamic Model of Worm Drives Machine. All stages of Mathematical Modelling are presents in follows:

- Identifications and assumptions problems for TSWM;
- Mathematical formulation of constraints and dynamic equations suitable for computing;
- Construction of Switching Function and Switching Algorithm;
- Simulation examples.

## Model Identification and Initial Assumptions

Configuration of TSWM (fig.1) is based on orthogonal complements in terms of partition co-ordinates [1,2] and under rigid-body approach.

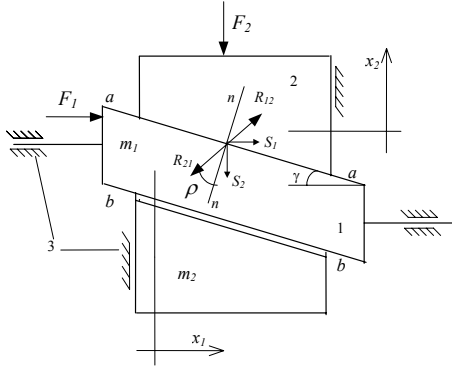


Fig.1 Design Dynamic Model

We start by making the following assumptions:

- TSWM is represent the Multibody System (MBS) with moving in orthogonal complement and interconnected rigid and inertial Wedge segments (bodies 1 and 2) by inclination surface;
- TSWM realized two different contact cases on inclination surface (a-a) or (b-b) with holonomic constraints due to existing clearance;
- We assumed that clearances size in TSWM is so small, that allows to accept a shock-free and instantaneous (zero in time) transitions from one contact case to another. Those transitions events characterise discontinuities on a stationary branch of model motion;
- The sliding friction forces during assembly motion of TSWM assumed only between Wedge segments on inclination surface correspond to frictional worm kinematics pair;
- Contacts between orthogonal fixed guides 3 and moving wedge segments (bodies) 1 and 2 are realised an ideal type of joints where friction forces are neglected.

Parameter Identification:

$x_1, x_2$  - joint co-ordinates (time-depended ) that attached to moving bodies 1 and 2 in orthogonal complements. They arbitrary separated on one independent (main) and dependent co-ordinates that needed to treat virtual assembly displacements of moving bodies;

$F_1, F_2$  - external actuated forces applied to bodies 1 and 2 along the co-ordinate directions  $x_1, x_2$  which assumed as known systems parameters;

$S_1, S_2$  - internal reduced reaction forces as the projections of reactions  $R_{12}, R_{21}$  which applied to bodies 1 and 2 along the co-ordinate directions  $x_1, x_2$  and assumed as unknown parameters. They formed in assembly a variable internal force

contour for each closed-loop contact-cases with corresponding Force Transfer Function (FTF);  
 $m_1, m_2$  - reduced masses of bodies 1,2 which are partitioned by contact. They indicate the inertia elements of Drives System that connected with the worm shaft and the shaft of a worm wheel respectively;  
 $\gamma$  - inclination angle of contact surface, which identify a lead angle of worm;  
 $\rho$  - frictional angle indicates the value of sliding friction forces, which dependent nonlinearly on the velocity of sliding  $V_s$ . The analytical description of  $\rho = f(V_s)$  may be presented by non-linear function  $\rho(\dot{x}_1) = 1/(a(\dot{x}_1/\cos\gamma)^b + c)$  [7], where  $V_s = \dot{x}_1/\cos\gamma$ ;  $a, b, c$  - coefficients, obtained from approximation of experimental data upon accounting mechanical properties of contact materials.

## Kinematic constraints formulation

The kinematic constraints equations of TSWM has generated from geometrical interpretation of contact collision between assembly moving bodies in orthogonal directions and written as

$$\frac{x_1}{x_2} = \frac{1}{tg\gamma} \text{ or } x_2 = x_1 tg\gamma \quad (1)$$

The velocity and acceleration constraint equations we obtained after double time derivation of Eqs.1 assuming contact as holonomic constraints

$$\dot{x}_2 = \dot{x}_1 tg\gamma \quad (2),$$

$$\ddot{x}_2 = \ddot{x}_1 tg\gamma \quad (3).$$

The constant value mark as  $u_w = 1/tg\gamma$  means stabile kinematic ratio between TSWMs moving parts.

## Force constraints and Discontinuity Kernel Formulation

We consider here two force plans represented on fig.2 and fig.3 which have describe the relationships between internal reduced forces  $S_1$  and  $S_2$  correspond to each contact cases based on kinetostatic analysis and equilibrium system state .

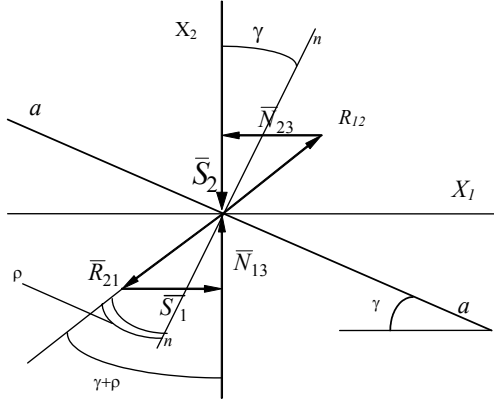


Fig.2. Force plan for active (a-a) contact surface

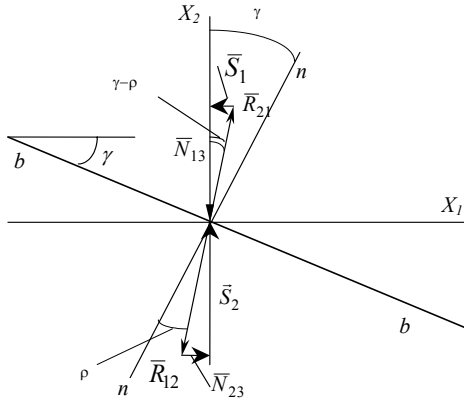


Fig.3. Force plan for active (b-b) contact surface

According to those plans we obtain force constraints written as :

$$\psi_1 = \frac{S_1}{S_2} = -tg(\gamma + \rho), \quad (4)$$

$$\psi_2 = \frac{S_1}{S_2} = -tg(\gamma - \rho), \quad (5)$$

where  $\psi_1$  - the Force Transfer Function (FTF) mark by index "1" for active contact-case (a-a) which identify the type of System structure with direct force/energy stream and undergoing conditions  $S_1 > 0, S_2 < 0$ ;  $\psi_2$  - the Force Transfer Function (FTF) mark by index "2" for active contact-case (b-b) which identify the type of System structure with inverse force/energy stream and undergoing conditions  $S_1 < 0, S_2 > 0$ . Each FTF describe analytically the level of effectiveness of internal force transformation for each active contact-cases.

The set of FTFs may be presents by one parameter  $\psi = \{\psi_j, j = \overline{1,2}\}$  and named the Discontinuity Kernel (DK) of Force constraints with corresponding components (5) and (6). The DK is piecewise continuous function which indicate ve

jump discontinuities, e.g. contact and force transitions during dynamic process.

In general form the Force constraint equation have been written as

$$S_1 = \psi S_2, \quad (6)$$

where  $\psi = \{\psi_j, j = \overline{1,2}\}$  - presented the Discontinuity Kernel of System which formulated by pair components - FTFs.

For estimation the level of increasing force stream on systems output we can use following coefficients for every contact-cases:  $K_1 = 1/|\psi_1|$  and  $K_2 = 1/|\psi_2|$ .

## Mathematical model and switching function formulation

The dynamic equations of motion are written in Newtons formulation together with constraints equations (3),(6) as DAEs :

$$\begin{aligned} m_1 \ddot{x}_1 &= F_1 - S_1, \\ m_2 \ddot{x}_2 &= F_2 - S_2, \\ \ddot{x}_2 &= \ddot{x}_1 tg \gamma, \\ S_1 &= S_2 \psi, (\psi = \{\psi_1(4), \psi_2(5)\}). \end{aligned} \quad (7)$$

These DAEs we are able to reduce to a minimal set, i.e. one equation per system degree of freedom, by algebraic eliminating of unknown internal constraint forces  $S_1, S_2$  and dependent acceleration coordinate selected as  $\ddot{x}_2$  from dynamic equations. The residual equation of motion with respect to independent co-ordinates  $x_1, x_1, \dot{x}_1$  and DK  $\psi$  in the right hand side we obtained in the form

$$\ddot{x}_1 = \frac{F_1 - F_2 \psi}{m_1 - m_2 \psi tg \gamma}, (\psi = \{\psi_j, j = \overline{1,2}\}), \quad (8)$$

with added initial systems conditions

$$x_1(0) = x_{10}, \dot{x}_1(0) = \dot{x}_{10}. \quad (8a)$$

The numerical treatment of problem (8),(8a) is complicated by the presence of DK that need to introduce some switching function and switching conditions in conjunction with standard step-by-step integration methods [5]. Thus we present the specially designed conditions for monitored systems DK by switching function

$$\psi = \begin{cases} \psi_1 & \text{for } \varphi(t) > 0, \\ \psi_2 & \text{for } \varphi(t) < 0. \end{cases} \quad (9)$$

where  $\varphi(t) = \Phi(t, x_1, \dot{x}_1)$  - time and state depended Switching Function (SF). The condition  $\varphi(t) > 0$  describe system structure and contact case type

with  $\psi = \psi_1$ ; the condition  $\varphi(t) < 0$  - respectively with  $\psi = \psi_2$ .

The conditions under which the discontinuity occurs on the time step  $[t_i, t_{i+1}]$  of current process can be formulated as

$$\varphi(\tau) = \Phi(\tau, x_1(\tau), \dot{x}_1(\tau)) = 0, \quad (10)$$

$$S_1(\tau) = S_2(\tau) = 0, \quad (11)$$

where  $\tau = \tau_k, (k = 1, 2, 3, \dots), \tau \in [t_i, t_{i+1}]$  denotes the Switching Points (SPs) if  $\varphi(t)$  changes its sign on the time scale. The quality (11) has satisfied instantaneous transition and critical state of system when contact-cases changes and internal forces  $S_1, S_2$  are deactivated that provide jump changes in the DK. Substituted (11) in (7) we can obtain the Switching Function of System as algebraic difference between "own" bodies accelerations

$$\varphi(t) = \ddot{x}_1^*(t) - \ddot{x}_2^*(t) - \frac{1}{\text{tg}\gamma}, \quad (12)$$

where  $\ddot{x}_1^*(t) = F_1(t)/m_1$  - denotes "own" acceleration of body 1;

$\ddot{x}_2^*(t) = F_2(t)/m_2$  - denotes "own" acceleration of body 2.

In general the developed Mathematical Model with Switching Function must be solved iteratively by using a continuous representation of the numerical solutions  $x_1(t), \dot{x}_1(t)$  of systems motion according to Switching Algorithm shown below.

### Switching Algorithm

For simulation discontinuities on integration time step by standard computer program we construct the Switching Algorithm, which is based on assumption of adjusting solution at the end of the step in which discontinuity was occurs and fix time  $\tau = t_{i+1}$  as Discontinuity Point. This assumed to be implicitly defined the time  $\tau$  as root of SF on step length.

In general, the switching algorithm works on one time step in following options :

1. Initialization:  $i=0, t_i=t_0, y_i=y_0, \varphi(t_i)=\varphi(t_0)$ .
2. Check sign ( $\varphi(t_i)$ ) and define DK  $\psi_j(t_i)$ .
3. Integration of one step  $t_i \rightarrow t_{i+1}$  using  $f(t, y, \psi_j)$ .
4. Check sign ( $\varphi(t_{i+1})$ ) on the end of step.
5. If no sign change, continue the integration otherwise localize the discontinuity at  $\tau = t_{i+1}$  if  $|\varphi(\tau)| < \varepsilon$ .

6. Change to the new DK as  $\psi_{j\pm 1}(t_{i+1})$  in the right hand side and restarting the integration on one step.

7. Check condition  $t_{i+1} < T_{end}$ .

We put here state vector as  $y = (x_1, \dot{x}_1)^T$ .

These Algorithm and Mathematical Model are implemented to Matlab ODE 15s solver in Multibody code and confirm by the simulation results.

### Simulation example and results

For simulation example we use following parameters of TSWM which correspond to lift machine (320 kg load capacity) with worm unit. Constant parameters:  $m_1=586$  kg.,  $m_2=10275$  kg,  $F_2=4774$  N,  $\gamma=5.71^\circ$ ,  $\text{tg}\gamma=0,1$  for worm gearing  $R_1=0,032$  m,  $R_2=0,128$  m and kinematic ratio  $u=40$ .

Variable parameters: electric motor mechanical characteristic as external force  $F_1(\dot{x}_1) = 2F_k(W + 1/W)$ ,  $V_c = 3,35$  m/sec,  $V_k = 2,07$  m/sec,  $F_k = 4375$  N;

Frictional angle

$$\rho(\dot{x}_1) = 1 / ((a\dot{x}_1 / \cos\gamma)^b + c), a = 0,174; b = 0,612; c = 0,14$$

The simulation results are presented on fig.4,5,6,7 and demonstrate the behaviour of internal forces and FTFs using solutions of systems state variables  $x_1(t), \dot{x}_1(t), \ddot{x}_1(t)$  and  $\psi(t)$ , where

$$S_1 = m_1\ddot{x}_1 - F_1(\dot{x}_1), S_2 = S_1 / \psi$$

The results are for two different load cases:

- 1)  $F_2 > 0$ ; 2)  $F_2 < 0$ .

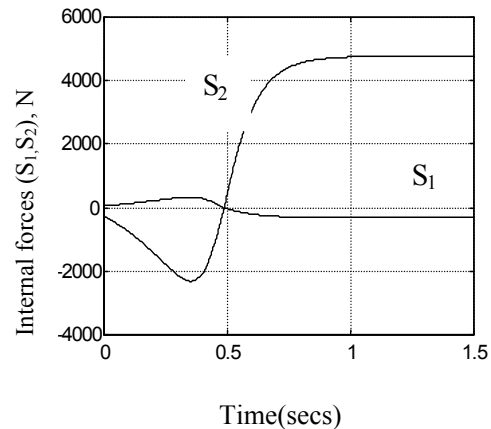


Fig.4 Simulation results for  $S_1, S_2$  in test case 1 with one switching point with states transition  $ST_1 \rightarrow ST_2$

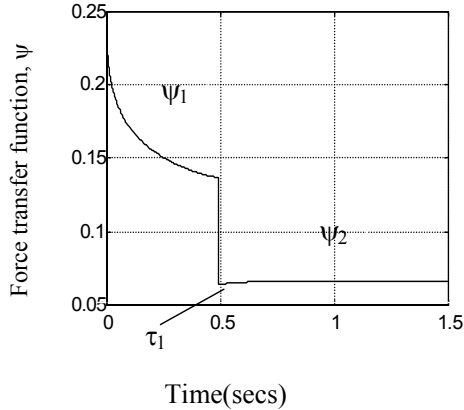


Fig.5. Simulation results for  $|\psi|$  in test - case 1,  $\tau_1 = 0,486$  sec - switching point

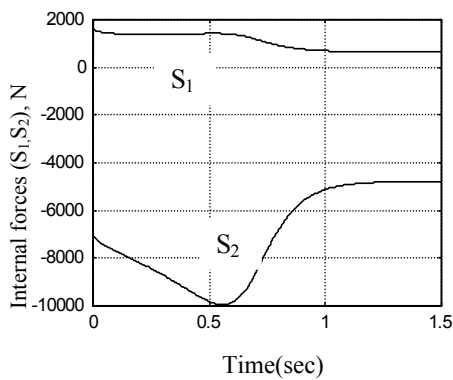


Fig.6 Simulation results for  $S_1, S_2$  in test-case 2 without switching points

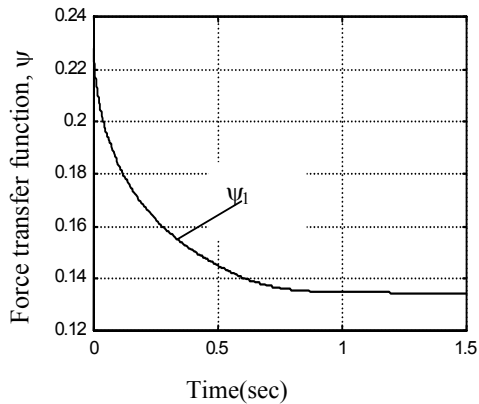


Fig.7 Simulation results for  $|\psi|$  in test-case 2

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## Conclusions

The developed Mathematical Model with switching conditions and discontinuous simulation technique for analysis internal force contour in TSWM with clearance presence shown good results and may be used not only for Worm Drives Machines but also useful for Machines with screw joints type.