Generation of Random Atmosphere with Application to Statistical Flight Simulation

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abstract- A simplifie method for mathematical modeling and simulation of atmospheric parameteres as a set of random functions of altitude has been developed. The proposed approach is based on canonical expansion method and eliminates complexity in coordinate functions determination.Results of simulation demonstrate that it may be easily computerized and used as a moudule of software for Monte Carlo flight modeling.

Keywords: stochastic atmosphere; statistical flight simulation; canonical expansion;

1. Introduction

An aerospace launch vehicle controlled motion is influenced by numerous random factors, such as deviation of motion initial conditions from nominal, parametere deviations of vehicle frame, the engine thrust, the control system, other systems from nominal values, external disturbances, acting on a vehicle due to Earth actual atmosphere and gravity field deviations from reference ones, noises of control system sensors,etc.

A launch vehicle actual trajectory deviates from nominal under the action of these random factors.

Deviation of the launch trajectory and terminal condition(satellite injection requirments) due to random disturbing factors may be so large that the mission task will be not executed.

The aim of the statistical analysis of the mission is to estimate statistical characteristics of vehicle motion.

All methods, that can be used for a priory statistical analysis of a system , can be separated in two groups[1],[2]:

-analytical methods,such as transition matrix method,spectral density transformation method,etc.

-statistical methods such as Monte Carlo simulation method.

In the Monte Carlo technique the artifical samples or data are generated by use of random number or randon function generators.

All random factors, which have to be taken into account, when a statistical model of the aerospace vehicle motion is developed, can be divided in to the following four sets :

Set 1. Random initial conditions

Random deviations of initial launch condions are referred to this set.

Set 2. Random events

Failues of vehicle components are included to this set.

Set 3. Random values

The following factors can be attributed to this set:

-aerodynamic coefficient deviation from nominal;

-mass and inertia parameters deviation from nominal;

-geometric misalignments;

All the above mentioned factors may be easily described as random varibles in the the Monte Carlo flight simulation.

Set 4. Random function of time or altitude

This set includes the following factors:

-wind in atmosphere;

-atmosphere density deviation from reference;

-propulsion system deviation;

Mathematical modeling and simulation of random functions and specially atmosphere is not so convenient and was subject of many investigatins[3],[4],[5].

To assess and predict the effect of atmosphere on the dynamic response and statistical characteristics of aerospace launch vehicle or aircraft, one needs a stochastic model of atmosphere. Such a model should recreate numerically parameteres of atmosphere as a random function of altitude.In the Monte Carlo flight simulation, the simulator of atmosphere should regenerate stochastic functions which satisfy adequately random characteristics of atmospheric parameters. Mathematical modeling and simulation of atmospheric parameteres as a stochastic functions fined many applications in aerospace and the others area in science and engineering [3],[5].

This research is aimed at mathematical modeling and implementation of a simplified stochastic generator of atmosphere for the Monte Carlo flight simulation.

2. Mathematical modeling

Random process modeling strongly depends on the statistical process characteristics. Basically two approaches can be used to simulate the Gaussian scalar continuous process [1],[2],[6]:

- The canonical expansion approach,
- The shaping filter approach,

The stochastic simulation algorithm based on the canonical expansion method is much simpler in implementation as against the shaping filter method [1]. Therefor in this work we developed a simplified alternative to canonical expansion method.

2.1 Canonical expansion method

Canonical expansion approach proposed by C.V.Poogachev, is a strong tool for random function modeling [1],[7]. This method proposes the following expansion for stochastic function:

$$x(t) = m(t) + \sum_{i=1}^{k} b_i j_i(t) \qquad (1)$$

Where:

$m(t)$ -Mean value of process (expected value),

$j_i(t)$ -Are deterministic functions (coordinate functions),

b_i - Are Gaussian mutually independent random variables.

In this paper random atmospere has been conditionally seprated in to the following two set:

Set 1. deviation of atmospheric parameteres (density, temperature, pressure).

Set 2 .atmospheric wind (wind velocity,wind azimuth).

Canonical expansion of atmospheric density, pressure and temperature may be selected as:

$$\Delta r(h) = \sum_{i=1}^{\infty} k_i j_{r_i}(h)$$
$$\Delta P(h) = \sum_{i=1}^{\infty} k_i j_{p_i}(h) \qquad (2)$$
$$\Delta T(h) = \sum_{i=1}^{\infty} k_i j_{T_i}(h)$$

Where:

$\Delta r, \Delta P, \Delta T$ - Random deviation of atmospheric density, pressure and temperature.

Canonical expansion of wind as a two dimensional stochastic vector-function may be considered as [8]:

$$v(h) = m_v + \sum_{i=1}^{k} b_i j_{vi}(h)$$
$$az(h) = m_{az} + \sum_{i=1}^{k} b_i j_{azi}(h) \qquad (3)$$

Where:

$v(h), az(h)$ – Random functions of altitude for wind velocity and wind azimuth,

$m_v(h), m_{az}(h)$ – Mean value of wind velocity and azimuth as a function of altitude,

j_{vi}, j_{azi} -Coordinate (deterministic) functions for wind velocity and azimuth,

The main problem in stochastic process simulation by the canonical expansion approach concerned with coordinate function determination [7][8]. This step requires heavy preliminary experimental data analyzing.

In order to find a simple algorithm for atmosphere modeling and simulation proposed a method based on combination of canonical expansion approach and concept of random vector. This method eliminates complexity of coordinate function determination.

3. The simplified generator of random atmosphere

To eliminate complexity in determination of coordinate functions $j_i(t)$ in this paper proposed a simplified alternative by using random vector concept as follow:

The Guassian vector $\vec{X}$ is described by the mean value vector m_X and the covariance matrix $K_X = |K_{ij}|_{n\times n}$.

In the general case, when $K_{ij} \neq 0$, the correlated Gaussian vector $\vec{X}$ can be simulated by linear transformation of the normalized Gaussian vector $\vec{b}$, consisting of the non-correlated normalized Gaussian variables $b_i(D_{b_i} = 1; i = 1,....n)$.This transformation is following:

$$\vec{X} = \vec{m}_X + A\vec{b} \tag{3}$$

The transformation matrix $A = |a_{ij}|_{n\times n}$ is a three-angle matrix:

$$A = \begin{bmatrix} a_{11} & 0 & .... & 0 \\ a_{21} & a_{22} & .... & 0 \\ ... & .... & .... & .... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{bmatrix}$$

Element a_{ij} of this matrix are defined from equation:

$$AA^T = K_X \tag{4}$$

The scalar form of matrix equation (4) may be represented as follow:

$$\begin{aligned} & k_{x_1x_2} = a_{11}^2; \\ & k_{x_1x_2} = a_{11}a_{12}; \\ & .... \\ & k_{x_1x_n} = a_{11}a_{n1}; \\ & k_{x_2x_2} = a_{21}^2 + a_{22}^2; \\ & .... \\ & k_{x_2x_n} = a_{21}a_{n1} + a_{22} + a_{n2}; \\ & .... \\ & k_{x_nx_n} = a_{n1}^2 + a_{n2}^2 + ... + a_{nn}^2; \end{aligned} \tag{5}$$

If we suppose stochastic wind as a discreet function of altitude (in altitudes $h_1, h_2,...., h_n$), it may be easily considered as a random vector. Therefore, it is no necessary to determine coordinate functions $j_i(h)$ in canonical expansion (2),(3).

Suppose that there are results of M atmospheric sounding test. But we need N realization of wind field for N flight test simulations ($N \succ\succ M$).

In order to regenerate $(N - M)$ stochastic wind realization in this work recommended following procedure:

1- Preliminary preparation of experimental data for selected altitudes ($h_1, h_2,...., h_n$);

2-Covariance matrix (K) calculation (for atmospheric parameteres, wind velocity and wind azimuth);

3- Matrix A calculation using equation (5);

4-Generation independent random vector $\vec{b}$ using standard programs;

5-Atmospheric Parameteres, Wind velocity and azimuth modeling using equations (2),(3),(4).

Block diagram of the proposed method illustrated in figure1.

3. Simulation example

The proposed algorithm for regeneration of (N-M) realizations of the stochastic atmospher with the given statistical characteristic is applied on limited number of experimental atmospheric measurement test (M=8). Some result of simulation for atmospheric density and pressure, wind velocity and azimuth as stochastic functions of altitude is presented in figures 1-4.In these figures sign + indicates density,pressur, wind velocity and azimuth generated by simplified model.

Conclusions

An simplified approach for stochastic atmospheric modeling based on canonical expansion and random vector generation is developed.

The result of simulations demonstrated capability and practical aspects of proposed approach especially in aerospace researches.

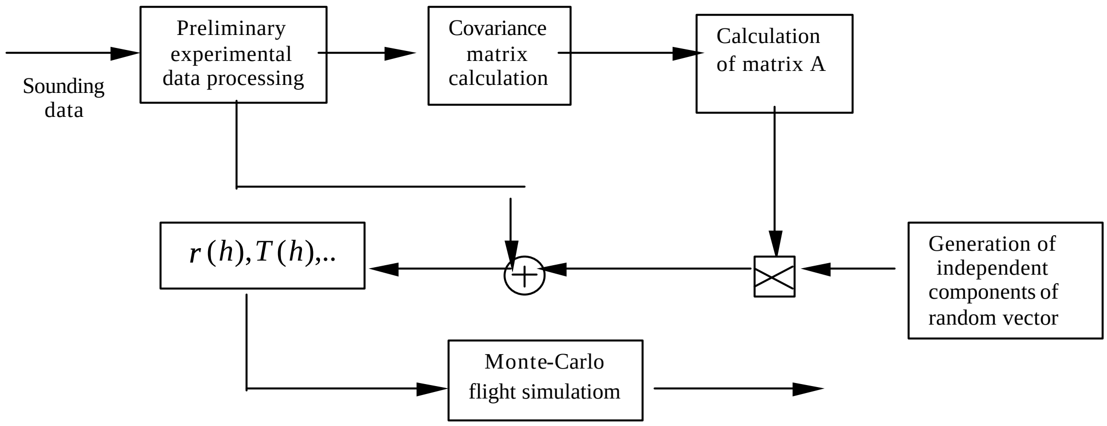
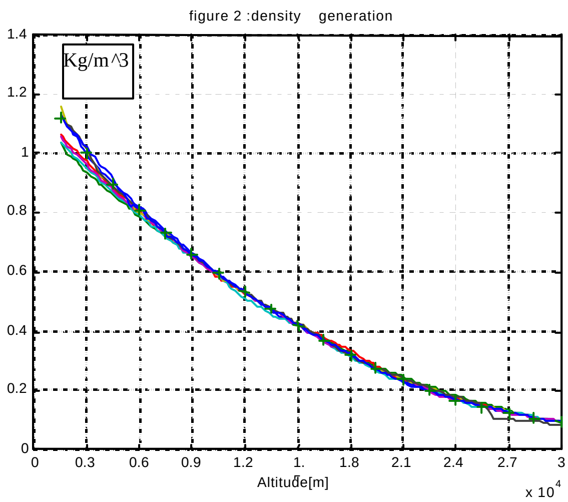
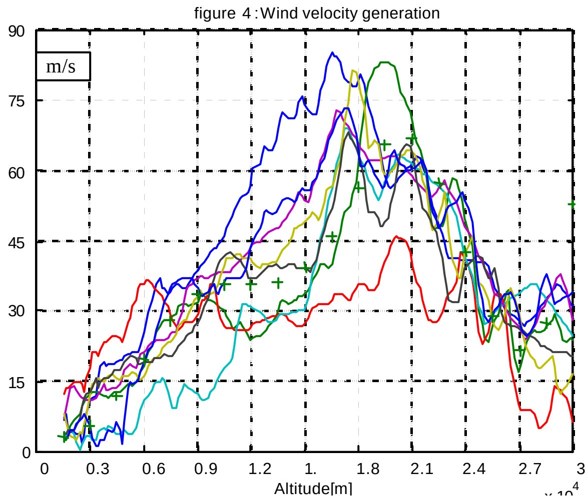
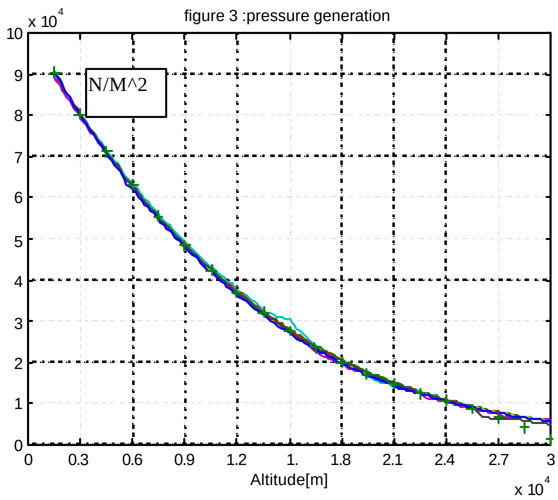
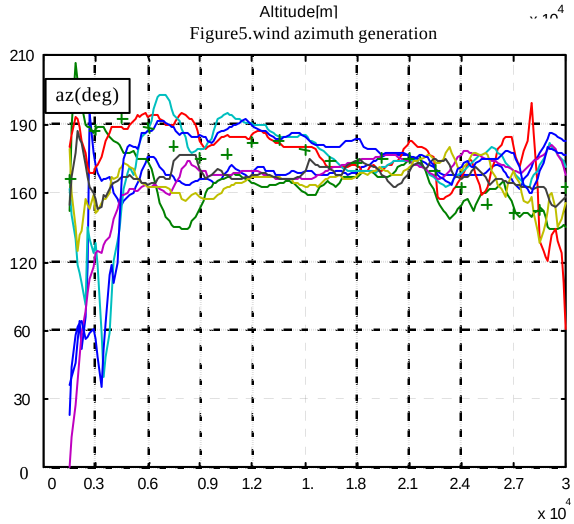


Figure1.Block diagram of simplified proposed method for stochastic atmosphere generation









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