Exploiting Dual Conditions in Economic Dispatch of District Heating Systems

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August 5, 2003

Abstract

The paper deals with the economic dispatch problem for a district heating system. The distribution network is modeled with a number of nodes and arcs, implying that time delays and restrictions in distribution capacity can be considered. A solution algorithm for the corresponding nonlinear optimization problem is proposed. The method is based on Lagrangian relaxation and exploits the relations between the optimal Lagrangian multipliers. The model is implemented and the performance of the algorithm is illustrated with some computational results.

Keywords: Lagrangian relaxation, district heating, economic dispatch.

1 Introduction

District heating is a technique used to supply cities with heat from a common heating system. Both from an economical and an environmental point of view, the technique has in many situations been found superior to other technologies. This has made district heating common in many countries, especially in countries with cold winters.

The heat, i.e. the hot water distributed in the district heating network, is produced in production units of different types, using different types of fuels. There can be units that are only producing heat, and Combined Heat and Power (CHP) units, which are units producing both heat and power. Often the system also includes one or several heat storages in which it is possible to store heat for later use. In some systems all production units are located in the same plant, but more common is that the units are located at different places in the network.

Typical decisions that must be taken due to the operation planning are e.g. when to start and stop the production units, and when to charge and discharge the heat storage. The problem may conceptually be divided into two subproblems: the unit commitment problem is the problem to determine when a unit shall be producing or not, and the economic dispatch problem is to find the optimal production levels given which units are producing. Typically a time horizon of one week, partitioned into one-hour intervals, is considered. The problem has much in common with the unit commitment and economic dispatch problems for power systems, which are well described in the literature [7], [8]. The main difference between the problem for a power system and a heating system is that the operation decisions for a district heating system must take in consideration the time delays in the distribution network. Since electrical power flows with significantly higher speed through the power grid, the corresponding is not relevant in power production planning.

The present paper deals with the economic dispatch problem for a district heating system. Section 2 defines the problem as a mathematical optimization problem. In the traditional way of modeling, see e.g. [1], [2], [3], [6], the production units are assumed to fulfill a pre-specified heat demand with heat produced at the same time it is consumed. The heat demand is normally described as one single demand curve, which implies that time delays, restrictions in distribution capacity in the network, and the fact that the consumers and the production units are located at different places in the network, are not handled. In the model presented in the paper, these features are considered.

In Section 3 a solution strategy based on Lagrangian relaxation is presented. The equations defining the energy balance at each node are combined with Lagrangian multipliers and relaxed, implying that the original (primal)

problem will decompose into a number of smaller subproblems that can be solved independently from each other. However, a dual problem appears and must be solved.

A solution algorithm for the dual problem is developed in Section 4. The method exploits the insight that there are specific relations between the optimal Lagrangian multipliers. The use of the relations between the multipliers in models for production planning is not new. Ravn et. al. [2], [5], exploited the relations in an algorithm for the solution of the economic dispatch problem with heat storage. Some parts of the algorithms presented in the present paper are generalizations of these algorithms. The corresponding was also discussed in [4], where necessary and sufficient optimality conditions for the dual of a specific version of the unit commitment problem were developed.

The model is implemented and the performance of the algorithms are in Section 5 illustrated with some computational results. Finally, Section 6 gives some concluding remarks.

2 Economic Dispatch Problem

The district heating network is modeled as a network with a number of nodes and arcs. The production units and the consumers supply points are located at the nodes. The arcs represent possible ways (pipes) to transfer heat.

Let N be the number of nodes defining the district heating network, let $K(n)$ be the set of production units that are located at node n, and let I be the number of time intervals within which the problem is to be solved. The length of time interval i is τ_i hours.

The decision variables of the problem are defined as follows. First, let $q_{i,k}$ be the heat produced in unit k in time interval i. The exchange of heat in time interval i between node n and the remaining network is described with the variable $q^n_{i,S}$. Finally, the energy transferred from node n to node m during time interval i is $e^{(n,m)}_i$. There are restrictions on the production levels, on the heat exchange and on the transferred energy, described as the inequality bounds,

$$\underline{q}_{i,k} \leq q_{i,k} \leq \overline{q}_{i,k}, \tag{1}$$

$$\underline{q}^n_{i,S} \leq q^n_{i,S} \leq \overline{q}^n_{i,S} \tag{2}$$

and

$$0 \leq e^{(n,m)}_i \leq \overline{e}^{(n,m)}_i, \tag{3}$$

respectively.

The distribution network is modeled with an energy balance equation at each node, for each time interval,

$$q^n_{i,S}\tau_i + \sum_{m=1}^{N} e^{(n,m)}_i - \sum_{m=1}^{N} \eta^{(m,n)}_i e^{(m,n)}_{i-1} = 0, \tag{4}$$

where $\eta^{(m,n)}_i$ is a scalar between zero and one that models the losses in the network. The initial and final energy levels are assumed known,

$$\begin{cases} e^{(n,m)}_0 = \underline{e}^{(n,m)}_0 \\ e^{(n,m)}_I = \underline{e}^{(n,m)}_I, \end{cases} \tag{5}$$

where the parameters $\underline{e}^{(n,m)}_0$ and $\underline{e}^{(n,m)}_I$ are the initial and final energy levels, respectively. We here notice that the variable $e^{(n,n)}_i$ represents the energy content in a heat storage located at node n.

The heat demand that must be fulfilled at node n in time interval i is

$$\sum_{k \in K(n)} q_{i,k} + q^n_{i,S} = q^n_{i,D}. \tag{6}$$

Here the parameter $q^n_{i,D}$ is the corresponding heat demand.

The cost of producing heat in a production unit is modeled using a second order polynomial,

$$c_{i,k} = \left(\alpha^2_{i,k}(q_{i,k})^2 + \alpha^1_{i,k} q_{i,k} + \alpha^0_{i,k}\right)\tau_i, \tag{7}$$

where the parameters $\alpha^j_{i,k}$, $j = 0, 1, 2$, reflect costs for fuels and taxes, and specifically for CHP units, also the income for produced and sold electricity. The equation (7) is defined as strictly convex, i.e. $\alpha^2_{i,k} > 0$. The cost associated with the heat exchange $q^n_{i,S}$ is understood as pumping costs,

$$c^n_{i,S} = \alpha^n_{i,S}(q^n_{i,S})^2\tau_i, \tag{8}$$

where $\alpha^n_{i,S} > 0$.

To summarize, define the economic dispatch problem as the following mathematical programming problem,

$$\begin{aligned} \min_{q,e} \; & \left[\sum_{i=1}^{I} \sum_{n=1}^{N} \sum_{k \in K(n)} \left(\alpha^2_{i,k}(q_{i,k})^2 + \alpha^1_{i,k} q_{i,k} + \alpha^0_{i,k}\right)\tau_i \right. \\ & \left. + \sum_{i=1}^{I} \sum_{n=1}^{N} \alpha^n_{i,S}(q^n_{i,S})^2\tau_i \right] \\ s.t. \; & q^n_{i,S}\tau_i + \sum_{m=1}^{N} e^{(n,m)}_i - \sum_{m=1}^{N} \eta^{(m,n)}_i e^{(m,n)}_{i-1} = 0 \\ & \sum_{k \in K(n)} q_{i,k} + q^n_{i,S} = q^n_{i,D} \\ & \underline{q}_{i,k} \leq q_{i,k} \leq \overline{q}_{i,k} \\ & \underline{q}^n_{i,S} \leq q^n_{i,S} \leq \overline{q}^n_{i,S} \\ & 0 \leq e^{(n,m)}_i \leq \overline{e}^{(n,m)}_i \\ & e^{(n,m)}_0 = \underline{e}^{(n,m)}_0 \\ & e^{(n,m)}_I = \underline{e}^{(n,m)}_I. \end{aligned} \tag{9}$$

In practice the number of variables and constraints in the problem may be quite large. This, together with the separable structure of the problem, motivates the use of solution algorithms based on Lagrangian relaxation.

3 Solution by Lagrangian Relaxation

Lagrangian relaxation is performed by introducing multipliers $\lambda = (\lambda_1^1, ..., \lambda_I^1, ..., \lambda_1^N, ..., \lambda_I^N)$. Combining these with (4) and adding to the objective in (9) gives the relaxed problem,

$$\begin{aligned}
&\Phi(\lambda) = \\
&\min_{q,e} \left[\sum_{i=1}^{I} \sum_{n=1}^{N} \sum_{k \in K(n)} \left(\alpha_{i,k}^2 (q_{i,k})^2 + \alpha_{i,k}^1 q_{i,k} + \alpha_{i,k}^0 \right) \tau_i \right. \\
&\quad + \sum_{i=1}^{I} \sum_{n=1}^{N} \alpha_{i,S}^n (q_{i,S}^n)^2 \tau_i + \sum_{i=1}^{I} \sum_{n=1}^{N} \lambda_i^n \\
&\quad \left. \left(q_{i,S}^n \tau_i + \sum_{m=1}^{N} e_i^{(n,m)} - \sum_{m=1}^{N} \eta_i^{(m,n)} e_{i-1}^{(m,n)} \right) \right] \\
&\quad s.t. \quad \sum_{k \in K(n)} q_{i,k} + q_{i,S}^n = q_{i,D}^n \\
&\qquad \underline{q}_{i,k} \le q_{i,k} \le \overline{q}_{i,k} \\
&\qquad \underline{q}_{i,S}^n \le q_{i,S}^n \le \overline{q}_{i,S}^n \\
&\qquad 0 \le e_i^{(n,m)} \le \overline{e}_i^{(n,m)} \\
&\qquad e_0^{(n,m)} = \underline{e}_0^{(n,m)} \\
&\qquad e_I^{(n,m)} = \underline{e}_I^{(n,m)},
\end{aligned} \tag{10}$$

where $\Phi(\lambda)$ is the dual objective function. The corresponding dual problem is

$$\max_{\lambda} \left[\Phi(\lambda) \right]. \tag{11}$$

Relative to (11), the problem (9) is called the primal problem. From the construction of the dual problem, it is understood that any value of the dual objective function defines a lower bound on the optimal function value of the primal problem. The intention in algorithms based on Lagrangian relaxation is to maximize this lower bound, i.e. solve the dual problem, and then, given the optimal dual variables λ^*, derive a solution of the primal problem from the corresponding solution of the relaxed problem. A solution algorithm for the dual problem (11) is presented in Section 4. In the remaining of this section, the solution of the relaxed problem (10) is discussed.

Given a set of dual variables, λ, the relaxed problem (10) will decompose into a number of independent subproblems, one for each node-interval pair (n, i). The minimizing $\widetilde{q}_{i,k}$, $k \in K(n)$, and $\widetilde{q}_{i,S}^n$ can be found independently of $e_i^{(n,m)}$ as the solution to

$$\begin{aligned}
&\min_{q} \left[\sum_{k \in K(n)} \left(\alpha_{i,k}^2 (q_{i,k})^2 + \alpha_{i,k}^1 q_{i,k} + \alpha_{i,k}^0 \right) \right. \\
&\qquad \left. + \alpha_{i,S}^n (q_{i,S}^n)^2 + \lambda_i^n q_{i,S}^n \right] \\
&\quad s.t. \quad \sum_{k \in K(n)} q_{i,k} + q_{i,S}^n = q_{i,D}^n \\
&\qquad \underline{q}_{i,k} \le q_{i,k} \le \overline{q}_{i,k} \\
&\qquad \underline{q}_{i,S}^n \le q_{i,S}^n \le \overline{q}_{i,S}^n,
\end{aligned} \tag{12}$$

which will be easy to solve, since (12) is a well structured problem and usually of small dimensions.

The minimizing $\widetilde{e}_i^{(n,m)}$ is given as

$$\begin{cases}
\text{if } \lambda_i^n > \lambda_{i+1}^m \eta_{i+1}^{(n,m)} & \text{then } \widetilde{e}_i^{(n,m)} = 0 \\
\text{if } \lambda_i^n = \lambda_{i+1}^m \eta_{i+1}^{(n,m)} & \text{then } 0 \le \widetilde{e}_i^{(n,m)} \le \overline{e}_i^{(n,m)} \\
\text{if } \lambda_i^n < \lambda_{i+1}^m \eta_{i+1}^{(n,m)} & \text{then } \widetilde{e}_i^{(n,m)} = \overline{e}_i^{(n,m)},
\end{cases} \tag{13}$$

where $\widetilde{e}_i^{(n,m)}$ can be chosen arbitrarily between limits as indicated when $\lambda_i^n = \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$. The insight that $\widetilde{e}_i^{(n,m)}$ in the middle option of (13) can be chosen as suited was for the economic dispatch problem with heat storage exploited in the algorithms presented in [2], [5]. The corresponding is also considered here, resulting in the following solution algorithm for the relaxed problem:

Step 1. Compute $\widetilde{q}_{i,k}$ and $\widetilde{q}_{i,S}^n$, $k \in K(n)$, $n = 1, ..., N$, $i = 1, ..., I$, from (12).

Step 2. Choose $\underline{x}_i^{(n,m)}$ and $\overline{x}_i^{(n,m)}$, $n = 1, ..., N$, $m = 1, ..., N$, $i = 1, ..., I-1$, as follows:
if $\lambda_i^n > \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$ then
let $\underline{x}_i^{(n,m)} = \overline{x}_i^{(n,m)} = 0$
if $\lambda_i^n < \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$ then
let $\underline{x}_i^{(n,m)} = \overline{x}_i^{(n,m)} = \overline{e}_i^{(n,m)}$
otherwise
let $\underline{x}_i^{(n,m)} = 0$ and $\overline{x}_i^{(n,m)} = \overline{e}_i^{(n,m)}$

Step 3. Compute $\widetilde{e}_i^{(n,m)}$, $n = 1, ..., N$, $m = 1, ..., N$, $i = 1, ..., I$, from (14).

First, in Step 1, problem (12) is solved for each node and time interval. In Step 2 the parameters $\underline{x}_i^{(n,m)}$ and $\overline{x}_i^{(n,m)}$ are chosen in accordance with (13). The parameter $\underline{x}_i^{(n,m)}$ is defined as the lower bound on $\widetilde{e}_i^{(n,m)}$ that is relevant for the current dual variables λ. Similarly $\overline{x}_i^{(n,m)}$ defines the upper bound. Finally, in Step 3, the variables $\widetilde{e}_i^{(n,m)}$ are chosen to fulfill (4) as closely as possible within the limits posed by (12) and (13). This is achieved by solving

the linear programming problem,

$$\begin{aligned}
\min_{e,s} \ & \left[\sum_{i=1}^{I}\sum_{n=1}^{N}\left(s_{i,n}^{+}+s_{i,n}^{-}\right)\right] \\
s.t. \ & \widetilde{q}_{i,S}^{n}\tau_i + \sum_{m=1}^{N} e_i^{(n,m)} - \sum_{m=1}^{N}\eta_i^{(m,n)} e_{i-1}^{(m,n)} = s_{i,n}^{+} - s_{i,n}^{-} \\
& \underline{x}_i^{(n,m)} \le e_i^{(n,m)} \le \overline{x}_i^{(n,m)} \\
& e_0^{(n,m)} = \underline{e}_0^{(n,m)} \\
& e_I^{(n,m)} = \underline{e}_I^{(n,m)} \\
& s_{i,n}^{+} \ge 0 \\
& s_{i,n}^{-} \ge 0.
\end{aligned} \tag{14}$$

Here $s_{i,n}^{+}$ and $s_{i,n}^{-}$ are slack variables.

4 Solving the Dual Problem

Since the dual problem (11) is non-smooth, solution methods for non-smooth optimization [9] must be used. To this group of methods belong e.g. subgradient methods and bundle methods. However, these are general methods aimed for generally defined problems. The solution methodology presented in this section is suited to the current problem structure. The algorithm is based on the insight that there are specific relations between the optimal Lagrangian multipliers (i.e. the dual variables).

The subgradient method is a simple and intuitive method for non-smooth optimization. In each iteration v of the algorithm, a new set of dual variables, $[\lambda]^{v+1}$, is computed from the previous one, $[\lambda]^{v}$, by performing a subgradient step,

$$[\lambda]^{v+1} = [\lambda]^{v} + \alpha^{v} g. \tag{15}$$

Here g is a subgradient and $\alpha^{v} > 0$ is a step length. The components g_i^n of the subgradients are for problem (11) computed as

$$g_i^n = \widetilde{q}_{i,S}^{n}\tau_i + \sum_{m=1}^{N} \widetilde{e}_i^{(n,m)} - \sum_{m=1}^{N}\eta_i^{(m,n)}\widetilde{e}_{i-1}^{(m,n)}, \tag{16}$$

where $\widetilde{q}_{i,S}^{n}$ and $\widetilde{e}_i^{(n,m)}$, $m = 1, ..., N$, are part of the current solution of the relaxed problem. The sequence of α^{v} is chosen such that $[\lambda]^{v}$ converges to λ^{*} when v increases. Such sequences can be defined rather easily, however, the corresponding convergence of $[\lambda]^{v}$ towards λ^{*} may be very slow.

From the interpretation of the Lagrangian multipliers, the conclusion is that when $\overline{e}_i^{(n,m)} > 0$ the two multipliers λ_i^n and λ_{i+1}^m are related. The relations are summarized as follows. The multiplier λ_i^n is interpreted as the marginal cost of the energy transferred from node n during time interval i. Assuming no losses, i.e. $\eta_i^{(m,n)} = 1$ in (4), and that $e_i^{(n,m)}$ is not on its bounds, i.e. $0 < e_i^{(n,m)} < \overline{e}_i^{(n,m)}$, the marginal costs at node n in time interval i and at node m in time interval $i+1$ must be equal. If they were not, and if the marginal cost was lower at node n than at node m, it would be more economical to increase the amount of energy sent from node n to m. Correspondingly, if the marginal cost was higher at node n, it should be more economical to decrease the amount of energy sent. Concerning losses and the energy transfer bounds will give the conditions already stated in (13). In particular (13) implies that when $e_i^{(n,m)} = 0$, then $\lambda_i^n > \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$, and when $e_i^{(n,m)} = \overline{e}_i^{(n,m)}$, then $\lambda_i^n < \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$.

By exploiting the relations between the optimal multipliers we define an algorithm, referred to as the Backwards Sequential Projection Algorithm:

Step 1. Given a step length $\alpha^{v} > 0$. Compute $[\lambda_I^n]^{v+1} = [\lambda_I^n]^{v} + \alpha^{v} g_I^n$, $n = 1, ..., N$.

Step 2. Perform Step 3 for $i = I-1, ..., 1$.

Step 3. Perform steps 4 to 7 for $n = 1, ..., N$.

Step 4. Compute $d_i^n = [\lambda_i^n]^{v} + \alpha^{v} g_i^n$.

Step 5. If $g_i^n > 0$ then

$$\text{let } [\lambda_i^n]^{v+1} = \min\left\{d_i^n, \min_{m\in M}\left\{\left[\lambda_{i+1}^m \eta_{i+1}^{(n,m)}\right]^{v+1}\right\}\right\}$$

$$\text{where } M = \left\{ m : \begin{array}{l} [\lambda_i^n]^{v} < \left[\lambda_{i+1}^m \eta_{i+1}^{(n,m)}\right]^{v+1} \\ \text{and } \overline{e}_i^{(n,m)} > 0 \end{array} \right\}.$$

Step 6. If $g_i^n < 0$ then

$$\text{let } [\lambda_i^n]^{v+1} = \max\left\{d_i^n, \max_{m\in M}\left\{\left[\lambda_{i+1}^m \eta_{i+1}^{(n,m)}\right]^{v+1}\right\}\right\}$$

$$\text{where } M = \left\{ m : \begin{array}{l} [\lambda_i^n]^{v} > \left[\lambda_{i+1}^m \eta_{i+1}^{(n,m)}\right]^{v+1} \\ \text{and } \overline{e}_i^{(n,m)} > 0 \end{array} \right\}.$$

Step 7. If $g_i^n = 0$ then compute $[\lambda_i^n]^{v+1}$ as follows:

$$\text{Let } M = \left\{ m : \begin{array}{l} [\lambda_i^n]^{v} = \left[\lambda_{i+1}^m \eta_{i+1}^{(n,m)}\right]^{v} \\ \text{and } \overline{e}_i^{(n,m)} > 0 \end{array} \right\}.$$

if $|M| = 0$ then

$$\text{let } [\lambda_i^n]^{v+1} = [\lambda_i^n]^{v}$$

otherwise

$$\text{let } [\lambda_i^n]^{v+1} = \frac{\sum_{m\in M}\left[\lambda_{i+1}^m \eta_{i+1}^{(n,m)}\right]^{v+1}}{|M|}$$

The algorithm is a generalization of the Backwards Sequential Projection Algorithm for the economic dispatch problem with heat storage developed by Ravn et. el. [2], [5]. As in the subgradient method, a suitable sequence of the step length α^{v} is chosen for Step 1. Ordinary subgradient steps are performed for the dual variables corresponding to time interval I. For the remaining variables,

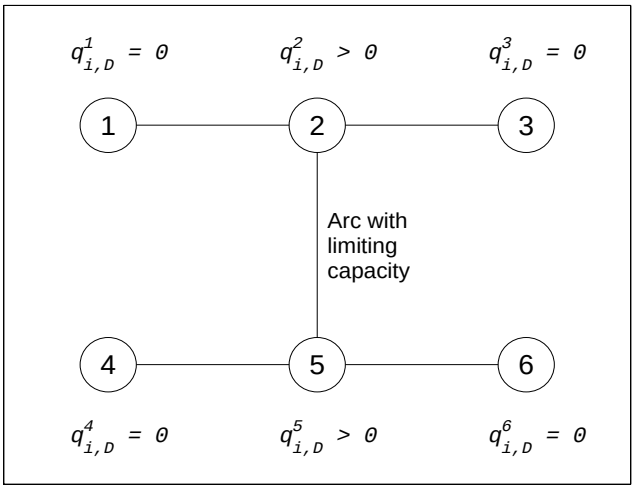

Figure 1: The network model.

the step length is restricted by the relations with the multipliers in consecutive nodes.

When the constraint (3) is identified as nonbinding and thereby is assumed to hold with strict inequalities, the relation $\lambda_i^n = \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$ can be substituted directly into the relaxed problem (10). The identification may be achieved by using a tolerance $\varepsilon > 0$. When $\left\| [\lambda_i^n]^v - \left[\lambda_{i+1}^m \eta_{i+1}^{(n,m)}\right]^v \right\| < \varepsilon$ is found to hold during a number of iterations, the relation is substituted for the relevant n, m and i. This implies that either λ_i^n or λ_{i+1}^m is eliminated from the relaxed problem. When λ_i^n is eliminated, the subgradient associated to λ_{i+1}^m is computed as

$$\widetilde{g}_{i+1}^m = \left(g_i^n \eta_{i+1}^{(n,m)} + g_{i+1}^m \right), \tag{17}$$

where g_i^n and g_{i+1}^m as before are computed from (16). Correspondingly, when λ_{i+1}^m is eliminated from the problem, the subgradient associated to λ_i^n is computed as

$$\widetilde{g}_i^n = \left(g_i^n + \frac{g_{i+1}^m}{\eta_{i+1}^{(n,m)}} \right). \tag{18}$$

We here notice that the substitution indeed may improve the algorithm, but an incorrect identification will also force the algorithm to converge to a non-optimal solution. The remedy here may be to during iterations also try to identify incorrectly defined nonbinding constraints.

5 Computational Results

This section illustrates the performance of the algorithms developed in Section 3 and 4. The input data used are derived from a district heating system in the region of Stockholm, Sweden. The model of the distribution network, illustrated in Figure 1, consists of six nodes, $N = 6$. The problem is solved over a time horizon of 24 hours, partitioned into half-hour intervals, i.e. $I = 48$ and $\tau_i = 0.5$, $i = 1, ..., I$. The system includes in total ten production units: three located at node one and four, respectively, and one located at each of the nodes two, three, five and six, i.e. $|K(1)| = |K(4)| = 3$ and $|K(2)| = |K(3)| = |K(5)| = |K(6)| = 1$. Of the ten units, two are aimed for base load production. These are located at the nodes one and four. The production units at the nodes three and six are mainly used in peak load situations. No heat demands are defined at the nodes one, three, four and six, $q_{i,D}^1 = q_{i,D}^3 = q_{i,D}^4 = q_{i,D}^6 = 0$, $i = 1, ..., I$. The demands at node two and five, i.e. $q_{i,D}^2$ and $q_{i,D}^5$, are during the planning horizon varying between 85 and 150 MW. Arc $(2, 5)$ has a limiting energy transfer capacity of 20 MW, i.e. $\overline{e}_i^{(2,5)} = 20$, $i = 1, ..., I$.

First ten initial iterations are performed by an ordinary subgradient method, cf. (15), and thereafter, the algorithm starts to use the Backwards Sequential Projection Algorithm to update the Lagrangian multipliers. From iteration 100 the version of the algorithm that identifies nonbinding inequalities and substitute the corresponding relation $\lambda_i^n = \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$ into the relaxed problem is applied. Here a tolerance $\varepsilon = 0.5$ is used.

The step length α^v is defined by

$$\alpha^v = \xi^v \frac{\Phi^* - \Phi([\lambda]^v)}{\|[g]^v\|}, \tag{19}$$

where Φ^* is the optimal dual objective. Equation (19) is known as *Polyak rule II*, which has proven to give robust results, see e.g. [9]. The choice for the sequence ξ^v is to start with $\xi^v = 1$ and then reduce ξ^v with a factor of two whenever $\Phi([\lambda]^v)$ has failed to increase in three iterations. In practice the optimal dual objective is unknown, and therefore normally a known upper bound of Φ^* is used instead. The dual objective for the first 200 iterations of the Backwards Sequential Projection Algorithm (Method 1) are shown in Figure 2. For illustrating purposes, the figure also shows the dual objective using the subgradient method not applying the Backwards Sequential Projection Algorithm (Method 2). From the figure we see that the pure subgradient method converges very slowly towards optimum. Since the relation $\lambda_i^n = \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$ will hold at the optimum for many of the dual variables, and a pure subgradient step in the dual variables not likely will match the condition exactly, the very slow convergence of the subgradient method is not unexpected. The Backwards Sequential Projection Algorithm, on the other hand, shows a better performance, especially when the

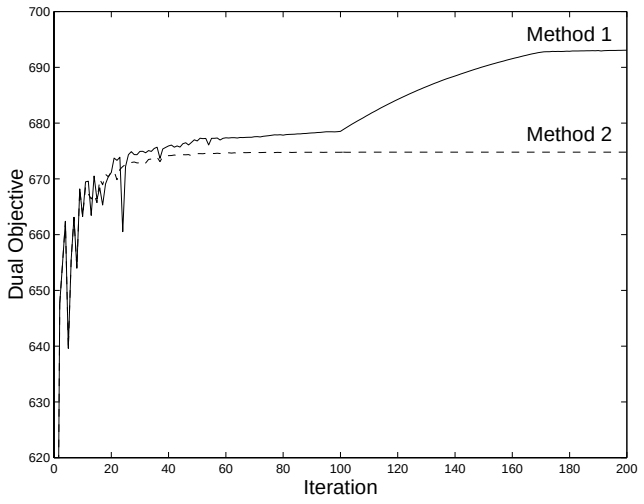

Figure 2: Dual objective during iterations using two different methods to compute $[\lambda]^{v+1}$.

substitution procedure is applied at iteration 100.

From the calculations we see that the new method (Method 1) solves the dual problem. We also see that the solution to the corresponding relaxed problem fulfills (4), which implies that the relaxed solution is an optimal solution of the primal problem.

A deeper analysis of the results verifies the relations between the optimal dual variables. Only in two time intervals the upper bound $\overline{e}_i^{(2,5)} = 20$ is restricting, which implies that $\lambda_i^n < \lambda_{i+1}^m \eta_{i+1}^{(n,m)}$ holds for the current i, n and m. At most other arcs, (m, n), the condition (3) is not limiting. In applications, heuristics may be used to identify at least some of these nonbinding conditions. The operators in the control rooms of the district heating systems often know which arcs (pipes) in the network that are not limiting.

6 Conclusions

The economic dispatch problem for a district heating system was considered and formulated as a mathematical nonlinear optimization problem. Contrary the traditional way of modelling, time delays and restrictions in distribution capacity in the network are considered. A solution algorithm bases on Lagrangian relaxation was proposed. The algorithm exploits the insight that there must be specific relations between the optimal Lagrangian multipliers. This was also verified with a numerical example. The relations are exploited both in the solution of the relaxed problem and in the updating of the multipliers. The computational results also indicate that the new method is superior the pure subgradient method aimed for general non-smooth optimization.

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